OPTIMAL REDUCTION OF EMISSIONS BY ABATEMENT

The aim of this chapter is to determine the desirable reduction in emissions, bearing in mind that emissions have a cost, but so does their reduction. This comparison of damage costs with abatement costs can be done for each individual emitter or for a set of emitters (e.g. in a region or country). The model is the same, but there are differences in the costs:

- A fairly large set of emitters can influence the marginal cost of damage, whereas this can be considered constant for an individual emitter
- The options for reducing emissions and therefore the magnitude of the costs are much greater for a group of emitters than for an individual emitter

Abatement measures include both technical measures (filters, material and energy substitution, efficiency gains) that reduce emissions at constant production and the reduction of production itself. The cost of reducing production is somewhat special, however, as it involves surplus losses for producers and consumers.

General principle of the optimal volume of emissions, respectively abatement

Consider a particular pollutant, such as SO_2 or CO_2, or harmful discharges into a natural environment, such as a river or lake. Various emitters contribute to these emissions, which cause damage. The boundary is defined in such a way that the geographical location of individual emitters is irrelevant. This means that the emissions of all emitters included in the perimeter contribute equally to the damage.[1] The damage depends on the total emissions of all these emitters:

$$\text{Damage} = D(E) \text{ with } E = \sum_i E_i$$

E_i represents the emissions from emitter i during the period under consideration. The length of this period should depend on the nature of the emissions, so that all emissions within this period contribute equally to the damage.[2]

1 If the pollution is very local, such as noise for example, a very limited perimeter should be defined, for example a neighbourhood, because reducing noise in one part of the city will not reduce the nuisance in another part of the city. If the pollution is global, on the other hand, such as the accumulation of greenhouse gases, the perimeter should include all the emitters on the planet.

2 If the pollution has a short lifetime, such as noise, then a very short time period should be defined, for example one hour, because reducing emissions at 10am will not compensate for excessive emissions at 10pm. If the damage depends, on the contrary, on the accumulation of emissions, such as climate change, then ideally a period of analysis of several years should be used, since emissions in 2022 contribute practically much to cumulative emissions as emissions in 2021.

The damage is assessed in monetary units. Sometimes, emissions can also have favourable effects on third parties.[3] These are subtracted from the damage.

In the vast majority of cases, the damage increases with emissions and the additional damage per unit of emission also increases. This is because the damage of an emission unit is higher if it is added to an already high level of emissions than if it is the only one:[4]

$$D(0) = 0,\ D'(E) \geq 0 \text{ and } D''(E) \geq 0$$

In the absence of government intervention, emitters do not take into account the damage caused by their emissions. They cause these emissions through their production or consumption, choosing the level of production or consumption and the patterns of production or consumption solely on the basis of their resources, the options available to them and the prices they have to pay or receive. This results in a level of emissions that we will call "*business as usual*" and note E^0. It also results in a level of damage without intervention equal to $D(E^0)$.

Abatement reduces emissions and therefore damage. For an abatement effort noted Δ_A, the residual emissions are:

$$E = E^0 - \Delta_A$$

$$\text{Damage with abatement} = D(E) = D(E^0 - \Delta_A)$$

It will be useful to consider the reduction in damage achieved by abatement as a "gain" from abatement. It is defined as follows:

$$\text{Gain from abatement} = G_A(\Delta_A) = D(E^0) - D(E^0 - \Delta_A)$$

The properties of the damage function imply the following properties of the abatement gain function:

$$G_A(0) = 0,\ G_A{}'(\Delta_A) \geq 0 \text{ and } G_A{}''(\Delta_A) \leq 0$$

This means that the gain increases with abatement effort, as the damage decreases, but it does so with a smaller and smaller increment the closer one gets to total elimination of emissions, since the first units of emissions cause relatively little damage. Figure 1 shows this, by reading the removal gain from right to left from point *a*.

3 Global warming saves on heating costs.

4 An apostrophe designates the first derivative of the function, which is interpreted as the increase in the function for a very small variation in its argument; here variation in damage D for a marginal variation in emissions E. Two apostrophes indicate the second derivative of the function, hence the derivative of the derivative. A curve with a negative second derivative "flattens". A curve with a zero second derivative is a straight line. A curve with a positive second derivative grows faster and faster.

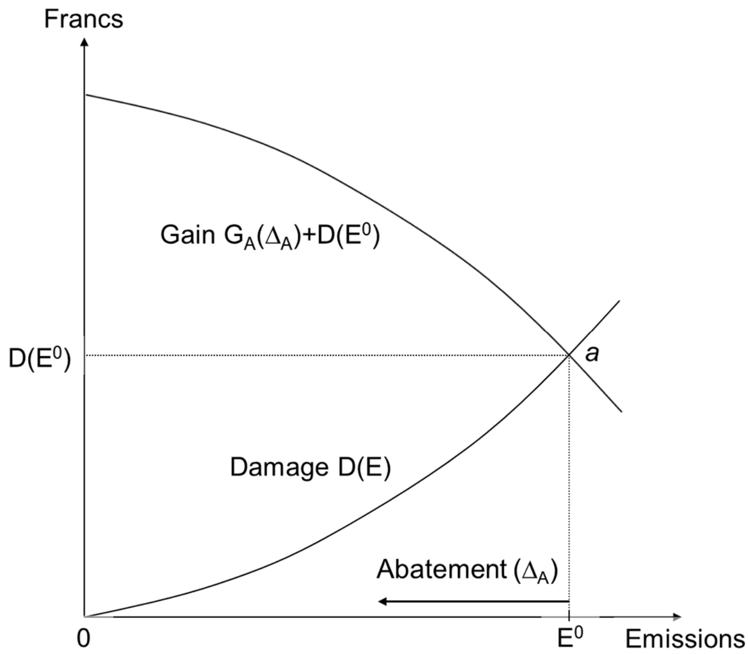

Figure 1: Damage and gain from abatement

Abatement usually has a cost, which depends on the amount of emissions avoided:

$$\text{Cost of abatement} = C_A(\Delta_A) \text{ with } \Delta_A = \sum_i \Delta_{Ai}$$

Δ_{Aj} represents the decrease in emissions of emitter j during the period considered. As each emitter bears its abatement costs individually, the total abatement cost is the sum of the individual abatement costs:

$$C_A(\Delta_A) = \sum_i C_{Ai}(\Delta_{Ai})$$

The abatement cost used here is a net cost, i.e. after deduction of material and energy savings for example.

In the vast majority of cases, the abatement cost increases with abatement effort and the additional cost per unit of abatement also increases. This is because emitters start by choosing the cheapest solutions for them and resort to more expensive solutions when they need to reduce their emissions even more. Hence:

$$C_A(0) = 0,\ C_A{}'(\Delta_A) \geq 0 \text{ and } C_A{}''(\Delta_A) \geq 0$$

The abatement cost has to be compared with the emission damage, or the abatement cost with the abatement gain, to determine the optimal level of abatement.

Optimal abatement effort

The table below summarizes the damage and abatement cost notation:

E	=	amount of harmful emissions
E^0	=	amount of emissions in the absence of any abatement efforts (*business as usual*)
Δ_A	=	abatement or quantity of emissions avoided: $E = E^0 - \Delta_A$
$D(E)$	=	total damage caused by emissions: $D(0)=0$, $D'>0$, $D''\geq 0$
$G_A(\Delta_A)$	=	gain from abatement, corresponding to the damage avoided: $G_A(\Delta_A) = D(E^0) - D(E^0 - \Delta_A)$, $G_A(0)=0$, $G_A'>0$ and $G_A''\leq 0$
$C_A(\Delta_A)$	=	abatement cost: $C_A(0)=0$, $C_A'\geq 0$, $C_A''\geq 0$

To determine the optimal abatement effort from the community's point of view, the total cost, which is the sum of the damage and abatement costs, must be minimised:

$$\min_{\Delta_A} D(E^0 - \Delta_A) + C_A(\Delta_A)$$

Using the definition of the abatement gain as the difference between the damage without abatement and the damage with abatement, the total cost minimisation programme becomes:

$$\min_{\Delta_A} \; D(E^0) - G(\Delta_A) + C_A(\Delta_A)$$

As the first term is fixed and constant, it is the same to maximise the net gain defined as the difference between the avoided damage and the abatement cost:

$$\max_{\Delta_A} G_A(\Delta_A) - C_A(\Delta_A) \qquad (1)$$

with the restriction that Δ_A must be between 0 and E^0. This problem, with the assumptions on the convexity of $C_A(.)$ and the concavity of $G_A(.)$, is represented in Figure 2.

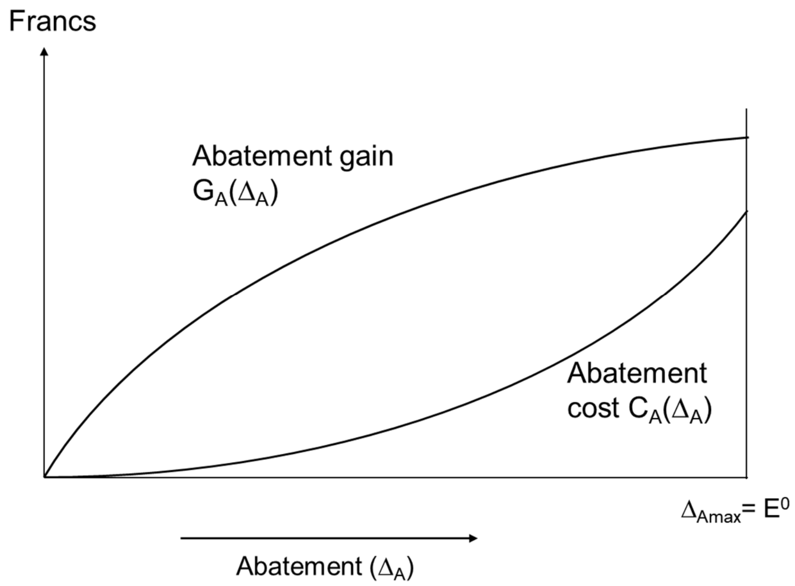

Figure 2: Gain and cost of abatement

The first-order necessary condition defining the optimal solutionΔ_A* can be written as follows:

$[G'(\Delta_A *)-C_A '(\Delta_A *)](E^0 -\Delta_A *)\Delta_A * = 0$

This equation describes three possible situations:

1. The marginal abatement cost is higher than the marginal gain for all values of Δ_A in the possible range $[0,E^0]$. In this case, Δ_A* = 0; the optimal abatement effort is zero (Figure 3 left).
2. The marginal abatement cost is lower than the marginal gain for all values of Δ_A in the interval $[0,E^0]$. In this case, Δ_A* = E^0 ; the optimal abatement effort removes all emissions (Figure 3 right).
3. If neither of these conditions is satisfied, the optimal effortΔ_A* lies within the interval $[0,E^0]$.

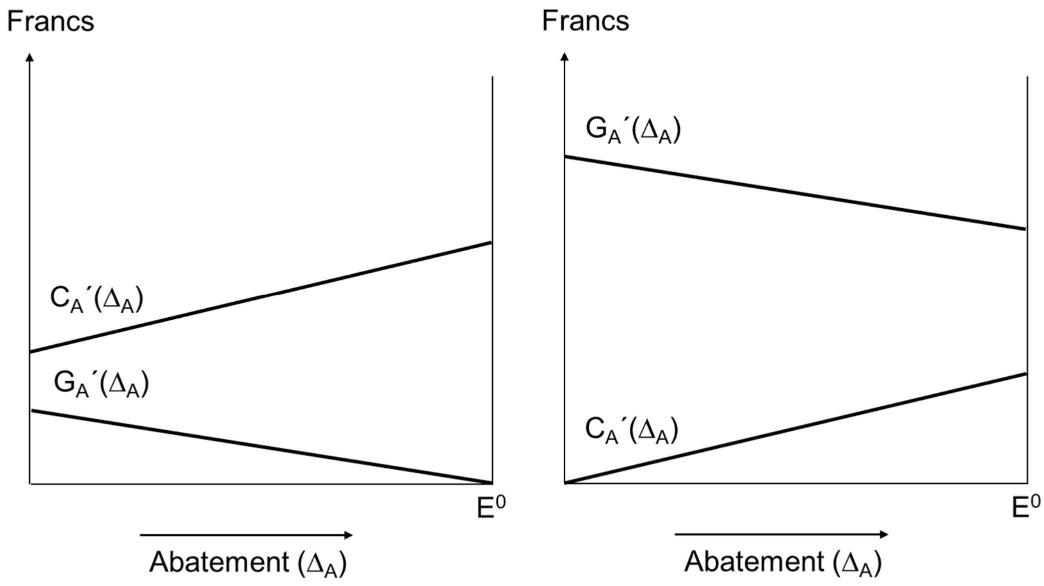

Figure 3: Optimal abatement zero (left) or total (right)

In the following, we will assume an inner solution: $0<\Delta_A^*<E$. This means that the marginal gain of the first abatement efforts exceeds their marginal cost – $G'(0) > C_A'(0)$ – and that the marginal gain of further abatement decreases as the marginal cost of abatement increases, the latter exceeding the marginal gain for an abatement effort below 100%, thus $G'(E^0) < C_A'(E^0)$.

The optimal abatement effort Δ_A^* will therefore now be defined by the condition:

$$G'(\Delta_A^*) = C_A'(\Delta_A^*) \qquad (2)$$

or[5]

$$D'(E^0-\Delta_A^*) = C_A'(\Delta_A^*) \qquad (3)$$

To decide on the optimal abatement effort Δ_A^*, the marginal gain from abatement must therefore be compared with the marginal cost of abatement. As long as the gain exceeds the cost, the abatement effort must be increased. This solution is shown in Figure 4.

5 Note that each derivative is calculated with respect to its argument, i.e. the marginal damage with respect to emissions E and the marginal abatement cost with respect to effort Δ_A.

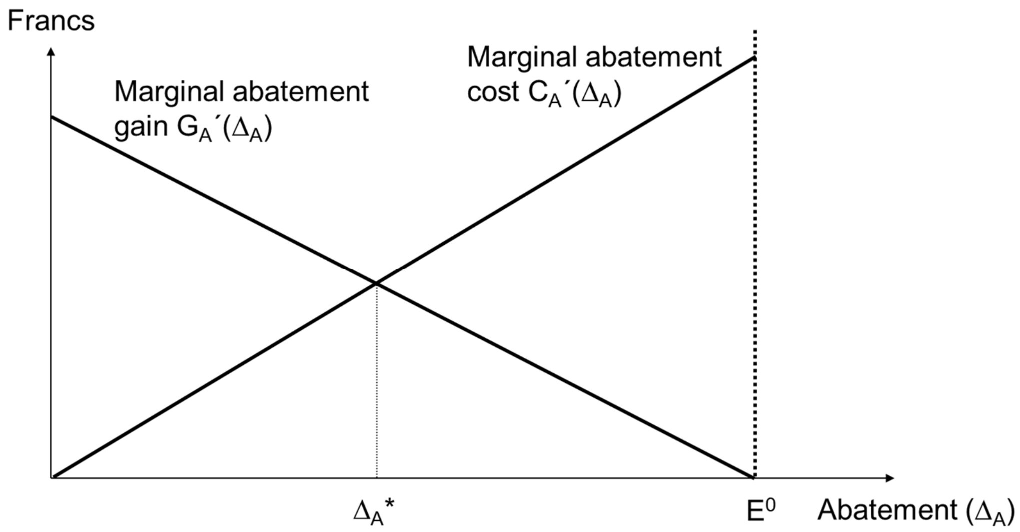

Figure 4: Socially optimal abatement

Figure 5 shows the total abatement cost (without fixed costs) imposed on emitters with the abatement effort Δ_A*, the net gain corresponding to the maximum of program (1) above, and the residual damage from the residual emissions E^0-Δ_A*. Geometrically, we see that Δ_A* is the value of Δ_A which not only maximises the net gain but also minimises the sum of the abatement cost and the residual damage.

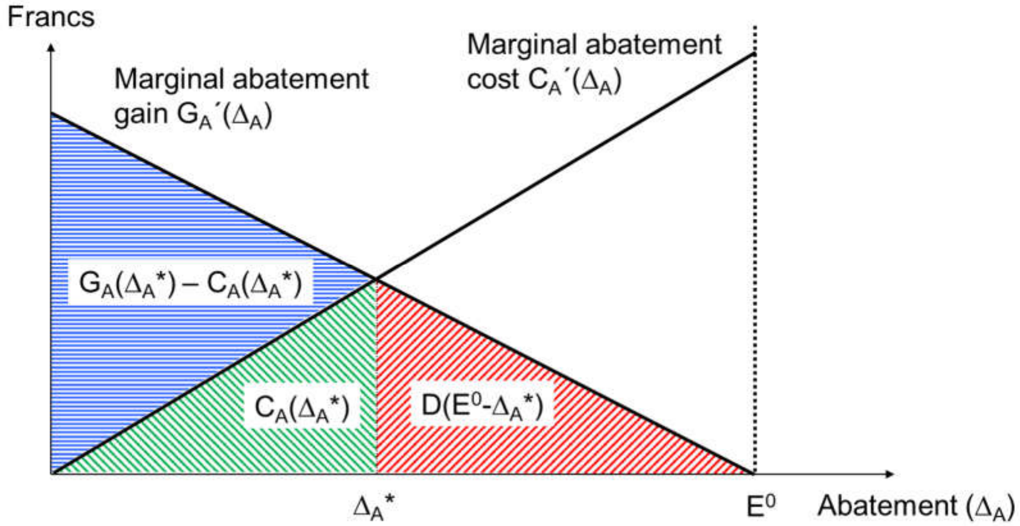

Figure 5: Optimal abatement cost, net gain and residual damage

Efficient distribution of abatement effort among several emitters

Total emissions are the sum of the emissions of N emitters indicated by i:

$$E = \sum_i E_i$$

Each emitter can reduce its emissions by abatement:

$$E_i = E_i^0 - \Delta_i$$

The damage and its reduction through abatement depend on the total emissions and the total abatement effort, i.e. the sum of the individual abatements. On the other hand, the total abatement cost is the sum of the abatement costs for the individual emitters. The optimisation programme (1) is therefore written:

$$\max_{\Delta_{Aj}} \; G_A\left(\sum_i \Delta_{Ai}\right) - \sum_i C_{Ai}(\Delta_{Ai})$$

The optimal abatement effortΔ_{Aj} * for emitter j therefore fulfils the following condition:

$$G_A'\left(\sum_i \Delta_{Ai}^*\right) = C_{Aj}'(\Delta_{Aj}^*) \qquad (4)$$

The marginal abatement cost for emitter j must be equal to the marginal gain from emission reductions. This marginal gain, the left-hand term in equation (4), is the same for all emitters. Therefore, condition (4) also implies that the marginal abatement costs of all emitters are equal:

$$C_{Aj}'(\Delta_{Aj}^*) = C_{Ak}'(\Delta_{Ak}^*)$$

The same condition of equality of marginal abatement costs is obtained by looking for the distribution of abatement efforts among N emitters that minimises the total abatement cost when a certain level of total abatement must be achieved (or a certain level of total emissions must not be exceeded):

$$\min_{\Delta_{Aj}} \; \sum_i C_{Ai}(\Delta_{Ai}) \quad \text{s.c.} \quad \sum_i \Delta_{Ai} \geq \bar{\Delta}_A$$